A SIMPLE CALCULATION SCHEME FOR THE LUMINOUS-FLAME EMISSIVITY

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A simple expression for the total emissivity of a luminous flame has been established on the basis of a flame-radiation model that consists of gray soot and non-gray gases. The gray-soot approximation is demonstrated to be an accurate characterization of soot emission from both the analytical formulation and the experimental observations. This simple model also allows a comparative analysis of soot emission characteristics in flames produced by various common solid and gaseous fuels. The present calculations for the luminous-flame emissivity show excellent agreement with existing experimental data and exact analysis.

1. Introduction

The understanding of thermal radiation from luminous flames is an important factor in the analysis of many engineering problems, such as heat transfer in fire, radiative exchange in a combustion chamber, etc. Many theoretical and experimental investigations of flame radiation have been reported in recent years.[1-8] Theoretically, the existing models predict the infrared emission from luminous flames utilizing the well-established wide-band theory for the radiating gases and the Mie theory for soot particles.[1,2] The practical application of these calculations, however, is limited. A calculation of the luminous-flame emissivity based on these models requires the values of all the basic flame parameters, i.e., the size distribution of soot, the volume fraction of soot, the optical properties of soot, the partial pressure of the participating gas and the flame temperature. A complete information of all these parameters in actual situations is generally not available.

Another problem with the existing theoretical models lies in the complexity of the exact expression for the total emissivity of luminous flames. Most of the existing experimental data are presented in the form of transmission and emission measurements,[3-5] and information about the various basic flame parameters must be deduced from these data. Based on the complicated exact expression, the reduction of data is extremely difficult. Consequently, experimental data are often interpreted with arbitrary empirical expressions with little physical basis, and there exists no correlation among data from different investigations. Generalization of the existing experimental results to application in an actual flame is almost impossible.

In this paper, a simple calculation scheme for the total emissivity of a homogeneous luminous flame is developed. Based on the available experimental data and physical consideration, the gray approximation is demonstrated to be sufficiently accurate for the calculation of soot emission. An effective soot emission parameter is introduced. For the gaseous components, non-gray band radiation is retained in the present model. The total emissivity of a homogenous luminous flame can then be expressed in a relatively simple form involving only five parameters, the soot emission parameter κ, the pathlength L, the mean flame temperature T_m, and the partial pressures of the two participating gases, H_2O and CO_2. This simple expression provides an effective and convenient basis to obtain the basic flame parameters for different fuels from flame emission data. The values of κ and T_m for some typical gaseous and solid fuels are tabulated. The contributions of soot emission to the total emissivity of the flames generated by solid fuels are found to be significantly different from those generated by gaseous fuels. The total emissivity of luminous flames generated by some typical polymer and gas-

eous fuels as calculated by this simple scheme shows excellent agreement with existing experimental data.

2. Analysis and Results

Soot Emission

Many experimental investigations have demonstrated that soot emission from luminous flames can be closedly correlated by the following gray expression,[3,4,5]

$$\epsilon_s = 1 - e^{-\kappa L} \tag{1}$$

where L is the physical pathlength of the flame, and κ is the effective soot-emission parameter independent of L. This gray-soot behavior in luminous flames has been observed by Markstein[3] for gaseous fuels, Buckius and Tien[4] for polymer fuels and Hägglund et. al.[5] for wood fuels. Explicit soot-emission data are provided only by Buckius and Tien and Hägglund et. al. Their results, together with the corresponding gray correlations, are presented in Fig. 1.

Equation (1) is also an excellent approximation to the following exact closed-form expression for the soot emissivity based on a non-gray analysis,[1,7]

$$\epsilon_s = 1 - \frac{15}{\pi^4}\psi^{(3)}(1+x) \tag{2}$$

with the definition,

$$x = \frac{cLT_m}{c_2} \tag{3}$$

and

$$c = 36\pi f_v \frac{n^2 k}{[n^2 - (nk)^2 + 2]^2 + 4n^2 k^2} \tag{4}$$

where c_2 is the Planck second constant, f_v is the volume fraction of soot particles and n and k are the infrared-average optical constants of soot particles. It is interesting to note that soot emission does not depend separately on the values of the fundamental soot parameters, f_v, n and k, but only depends on the value of the effective soot-concentration parameter c. Sato and Matsumoto[7] have demonstrated that an exponential function gives close agreement to Eq. (2) for small value of x. An exact numerical comparison, however, shows that Eq. (1) is an excellent approximation to Eq. (2) for all values of x provided that κ is given as

$$\kappa = 3.6\frac{cT_m}{c_2} \tag{5}$$

The comparison of the exact and approximate expressions for ϵ_s is illustrated in Fig. 2, and the agreement is within 7 percent.

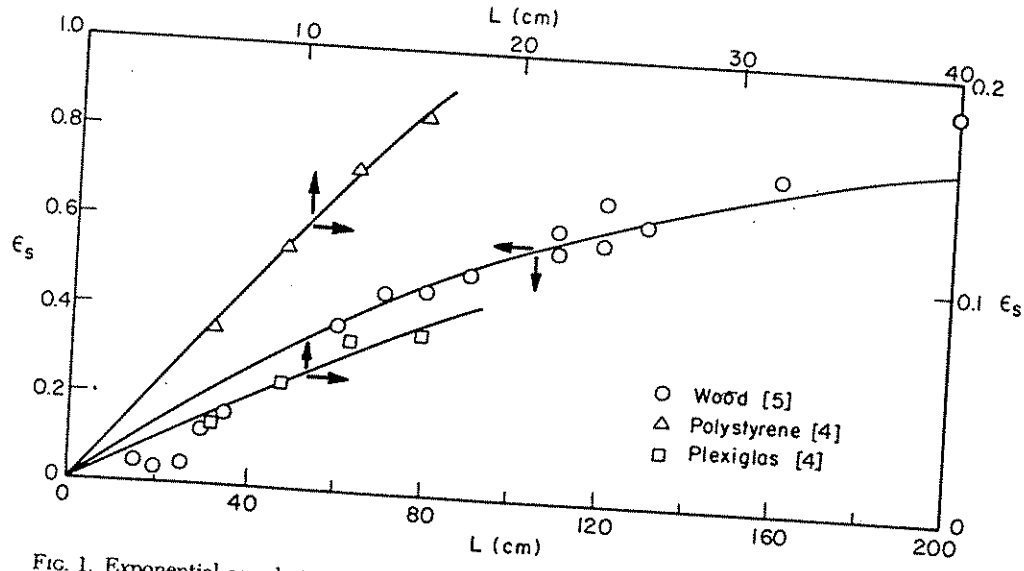

FIG. 1. Exponential correlation $(1 - e^{-\kappa L})$ of experimental data of soots emissivity for solid fuels.

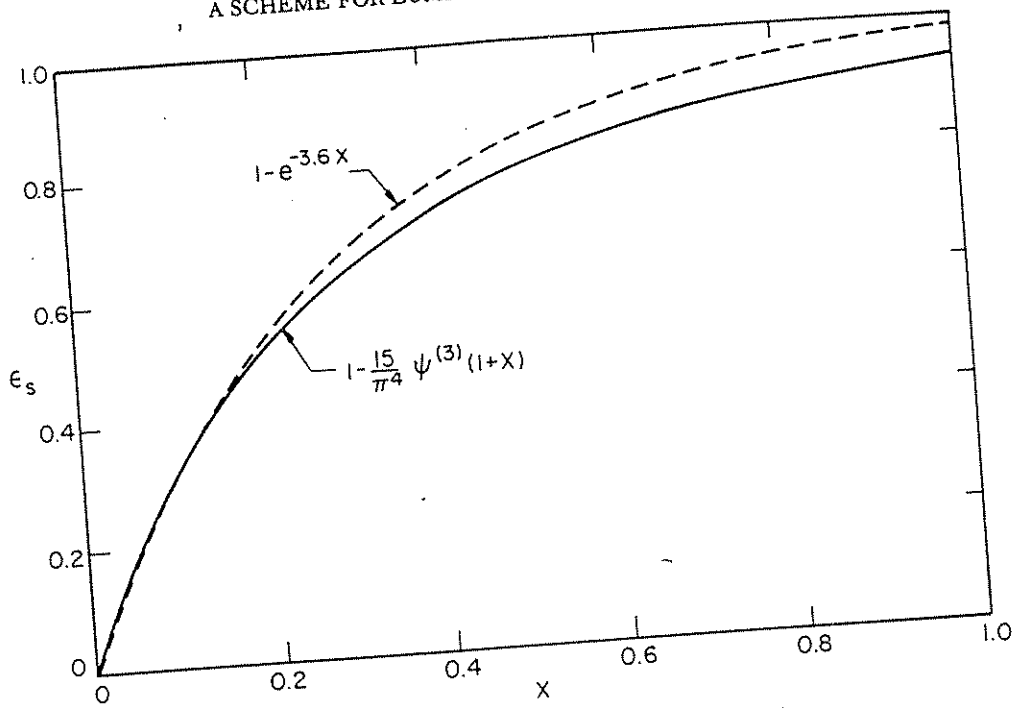

FIG. 2. Comparison of exact and approximate expressions for ϵ_s.

Physically, it is not too surprising that soot emission in luminous flames can be approximated effectively with the gray model. The absorption coefficient of soot is well known to depend on wavelength λ according to a $\lambda^{-\alpha}$ relationship. Theoretically, the value of α is expected to be unity based on the Rayleigh limit of small particulate absorption. This inverse wavelength variation ($\alpha \approx 1$) of the soot absorption coefficient has also been observed in many experimental investigations[9-11] under different burning conditions and for different fuels. The spectral emissivity of soot particles is thus large at the short-wavelength region, but small at the long-wavelength region. At the relatively high flame temperature (typically 1500°K), the blackbody emissive power is dominated by the radiant energy of short wavelength. It is reasonable to expect that soot emission, which is the product of the soot spectral emissivity and the blackbody emissive power, to be concentrated in a relatively narrow wavelength band in the short-wavelength region. The gray behavior of soot emission should therefore be observed.

Based on the gray approximation, calculation of the soot emissivity in a luminous flame is greatly simplified. Soot emission is now completely characterized by the effective soot-emission parameter κ, and the mean flame temperature T_m. Unlike the other fundamental soot parameters such as size distributions, volume fraction and optical properties, which are difficult to measure even under the most idealized experimental situation, the effective soot-emission parameter κ and the mean flame temperature T_m can be easily extrapolated from the spectroscopic measurement of flame emission[4,5] or the total flame-transmission measurement.[3] Using Eq. (5), the value of the effective soot-concentration parameter, c, can also be readily obtained. The values of κ, c and T_m for some commonly used gaseous and solid fuels are tabulated from experimental data[3,4,5] and presented in Table I.

It is interesting to note that the effective soot-concentration parameter, c, for gaseous fuels is one order of magnitude larger than that for solid fuels. These results suggest that physically, the volume fraction of soot particles in flames generated by gaseous fuels is much larger than those generated by solid fuels. There is also a general trend of increasing κ and c with the increase of the C/H ratio in various gaseous fuels.

Luminous-Flame Emissivity

The total emissivity of a luminous flame based on a non-gray analysis is given by[1]

TABLE I
Values of κ, c, and T_m for various fuels tabulated from experimental data.[3,4,5]

		C/H	$\kappa(cm^{-1})$	c	$T_m(^oK)$
Gas fuels	methane, CH_4	0.25	0.0645	2.00×10^{-5}	1289
	ethane, C_2H_6	0.33	0.0639	1.60×10^{-5}	1590
	ethylene, C_2H_4	0.50	0.1192	2.76×10^{-5}	1722
	propane, C_3H_8	0.38	0.1332	3.41×10^{-5}	1561
	isobutane, $(CH_3)_3CH$	0.40	0.1681	4.32×10^{-5}	1554
	n-butane, $(CH_3)(CH_2)_2(CH_3)$	0.40	0.1259	3.12×10^{-5}	1612
	propylene, C_3H_6	0.50	0.2407	6.46×10^{-5}	1490
	isobutylene, $(CH_3)_2CCH_2$	0.50	0.3072	8.70×10^{-5}	1409
	1,3-butadiene, $CH_2CHCHCH_2$	0.66	0.4542	1.35×10^{-4}	1348
Solid fuels	wood		0.008	1.8×10^{-6}	1732
	plexiglas, $(C_5H_8O_2)_n$	0.63	0.005	1.3×10^{-6}	1538
	polystyrene, $(C_8H_8)_n$	1.00	0.012	3.2×10^{-6}	1486

$$\epsilon_T = \frac{\pi}{\sigma T_m^4} \int_0^\infty I_{b\nu}(1 - \tau_{\nu p})\, d\nu + \frac{\pi}{\sigma T_m^4} \sum_{ij} \bar{\tau}_{\nu ijp}\, \bar{I}_{b\nu ij}\, A_{ij} \quad (6)$$

where $I_{b\nu}$ is the blackbody intensity, $\tau_{\nu p}$ is the transmissivity for soot particles, ij denotes the ith band of the jth gas species, A_{ij} is the total band absorptance, and the bar quantities indicate appropriate values taken over a gas band. On the basis of a gray-soot approximation, Eq. (6) is reduced to the following simple form

$$\epsilon_T = (1 - e^{-\kappa L}) + e^{-\kappa L} \epsilon_g \quad (7)$$

where ϵ_g is the emissivity of the radiating gases alone, calculated easily from the wide-band theory. The total emissivity of luminous flames predicted by Eq. (7) gives excellent agreement with the exact values predicted by Eq. (6). Some typical comparisons between the two expressions are presented in Table II.

The advantage of using Eq. (7) to predict the luminous-flame emissivity is that it only requires the values of five parameters, the effective soot emission parameter κ, the mean flame temperature T_m, the pathlength L and the partial pressures of the two participating gases, H_2O and CO_2. Even though a theoretical prediction of these parameters is still somewhat difficult to obtain, these parameters can all be easily measured. Equation (7), together with the tabulated values of these five parameters, thus provides a convenient method to estimate and compare the relative flammability of different fuels.

Comparison between Eq. (7) and the existing experimental data is difficult because in most experiments, not all the values of the five flame parameters are reported. Comparison is possible only with the data obtained by Markstein[3] and Buckius and Tien.[4] Assuming that the partial pressures for H_2O and CO_2 can be closely approximated by the stoichiometric partial pressures, comparison between the experimental and the theoretical value of the total intensity, I_T, for some typical polymer and gaseous fuels are presented in Fig. 3 and Table III. The agreement is excellent.

3. Conclusion

The gray approximation is shown to be accurate in the formulation of infrared emission from soot particles in a luminous flame. Based on the gray expression for soot emissivity, the values of the effective soot-concentration parameter and the mean flame temperature for some commonly used gaseous and solid fuels are extrapolated from existing experimental data. The volume fraction of soots in flames generated by gaseous fuels is found to be much larger than that generated by solid fuels.

A relatively simple expression for the total emissivity of a luminous flame is developed. Based on this simple expression, flame emission data can be easily interpreted to give

TABLE II

Approximate value of flame emissivities for some typical cases. (Values in parenthesis are based on exact calculations.)[1,2]

$T = 1000°K$	$P_{CO_2} = 0.09$	$P_{H_2O} = 0.09$	$T = 1800°K$	$P_{CO_2} = 0.1$	$P_{H_2O} = 0.1$
$f_v \times 10^4$ L(cm) \ L(cm)	10	100	ρL (kg/m^3)m \ $P_{CO_2}L$(atm-m)	0.002	0.15
	$\epsilon_g = 0.087$	$\epsilon_g = 0.243$		$\epsilon_g = 0.02$	$\epsilon_g = 0.2$
	ϵ_T	ϵ_T		ϵ_T	ϵ_T
0.0010	0.088 (0.088)	0.244 (0.244)	0.00001	0.032 (0.033)	0.21 (0.21)
0.0045	0.092 (0.093)	0.247 (0.248)	0.00005	0.080 (0.076)	0.25 (0.24)
0.024	0.114 (0.120)	0.265 (0.272)	0.0001	0.14 (0.13)	0.30 (0.28)
0.100	0.194 (0.218)	0.332 (0.356)	0.0005	0.48 (0.31)	0.58 (0.54)
0.310	0.381 (0.402)	0.486 (0.511)	0.001	0.72 (0.64)	0.78 (0.71)
1.000	0.739 (0.672)	0.783 (0.730)	0.002	0.92 (0.83)	0.94 (0.90)
3.000	0.979 (0.914)	0.982 (0.926)	0.005	0.99 (0.95)	0.99 (0.99)

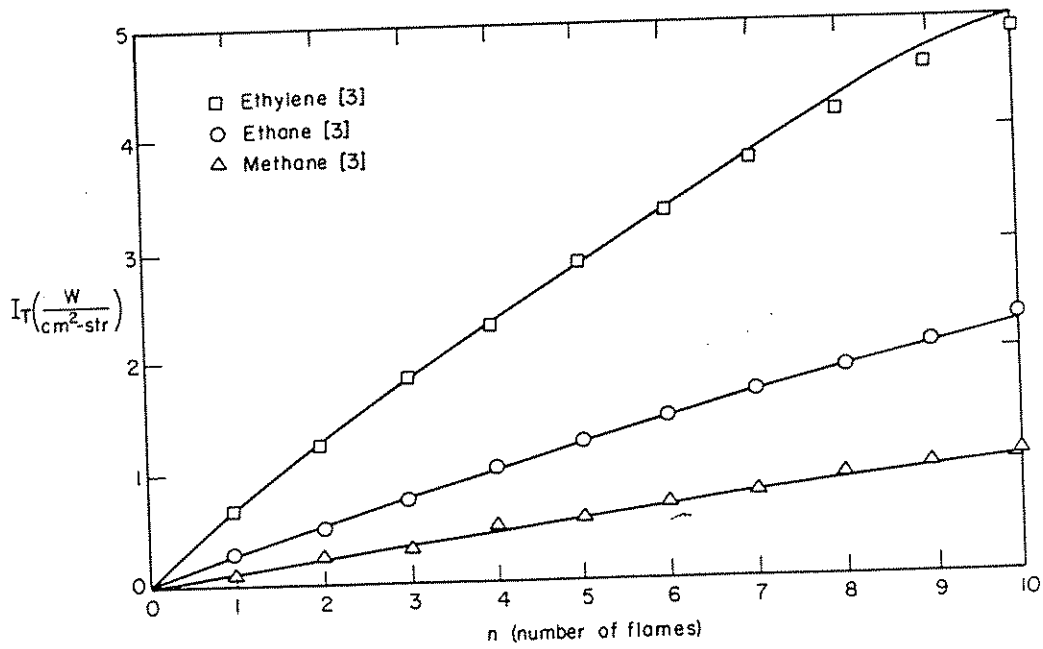

FIG. 3. Comparison between experimental and theoretical prediction of the luminous-flame emission of three typical gaseous fuels.

TABLE III
Theoretical prediction of luminous flame emission of two polymer fuels (values in parenthesis are measured experimentally.)[4]

	Polystyrene	Plexiglas
L(cm)	I_T(w/cm²-str)	I_T(w/cm²-str)
6.35	0.916 (0.767)	0.642 (0.528)
9.52	1.27 (1.12)	0.867 (0.799)
12.69	1.59 (1.46)	1.08 (1.03)
15.87	1.90 (1.66)	1.27 (1.15)

values of important flame parameters. The accuracy of this expression is demonstrated by comparison with the exact analytical calculation and the experimental data.

Nomenclature

- A total band absorptance, Eq. (6)
- c effective soot concentration parameter defined by Eq. (4)
- c_2 Planck's second constant, Eq. (3)
- f_v volume fraction of soot particles, Eq. (4)
- I radiative intensity, Eq. (6), Fig. (3), Table (3)
- k, n optical properties of soot particles, Eq. (4)
- L pathlength
- T temperature
- x dimensionless parameter defined by Eq. (3)
- $\Psi^{(3)}$ pentagamma function, Eq. (2)
- κ effective soot emission parameter, Eq. (5)
- ϵ emissivity
- σ Stefan-Boltzmann constant
- τ transmissivity
- ρ density of soot, Table 2

Subscripts

- b blackbody value
- g gas value
- ij ith band of the jth gas species
- m mean value
- p particle value
- s soot value
- T total value
- ν frequency

Acknowledgment

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COMMENTS

K. A. Bueters, Combustion Engineering Inc., USA. The authors have arrived at a result of primary importance and power. I am particularly delighted that they have arrived, from a wholly different basis, at a formulation identical in form to one I published in March 1974 in Combustion magazine, "Combustion Products Emissivity by F_E Operator." That is to say, the Yuen-Tien result is in one-to-one correspondence with the F_E relation, or

$$\frac{e^{KL} - 1 + Eg}{e^{KL}} \leftrightarrow \frac{F_E - 1 + \epsilon_H}{F_E}$$

which relation is in daily use at Windsor. If we grant the empirical validity of the gray gas law, then the Yuen-Tien relation on a, to me, sufficiently rigorous basis, shows that F_E and e^{KL} are one and the same. It would be remiss on my part not to also mention that the F_E relation, initially obtained in 1965, was shown on a gray-gas basis by R. H. Essenhigh in 1971 (see above paper). In conclusion, the two results have implications which are well worth further exploration and have great engineering power.

Authors' Reply. We wish to thank Dr. Bueters for bringing to our attention his 1974 paper, in which he arrived at a similar form through the use of an empirical factor.

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Ashok T. Modak, Factory Mutual Research, USA. I would like to first, make a comment and then ask a question. For luminous flames of most plastic materials the expression developed in this paper, for flame emissivity, can be simplified even further by neglecting the non-gray term due to gaseous radiation. Recent calculations,[1] in which the flames were assumed to be homogeneous and gray emitters, have shown excellent agreement with measured radiative energy fluxes to burning fuel surfaces and to targets external to fires of plastic materials. The gray assumption deteriorates for non-luminous flames e.g., of polyoxymethylene (Delrin).

Measurements at Factory Mutual[2] have shown that the flame absorption coefficient for polyoxymethylene, polymethyl methacrylate and polystyrene increases in that order. Is the relative ordering of your experimental results different? If so, why?

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2. Markstein, G. H., "Radiative Properties of Plastics Pool Fires," to be published.

Authors' Reply. The success of the gray assumption in the calculation of the luminous-flame emissivity for some plastic materials does not contradict the conclusion reached in this paper. In fact, Dr. Modak's observation concerning the gray assumption is completely consistent with equation (7), the general expression for the luminous-flame emissivity. For most plastic materials which generate very sooty flames, the second term in equation (7) is small. The expression for the luminous-flame emissivity therefore reduces to the gray-approximation solution. For some plastic materials, such as polyoxymethylene (i.e., Delrin), which generate rather non-sooty flames, the second term in equation (7) is not small. The gray assumption is clearly violated.

Measurements made by Buckius and Tien[4] demonstrated that the soot absorption coefficient for the three plastics in question increases in the following order: polymethyl methacrylate (i.e., Plexiglas), polyoxymethylene, polystyrene, with the first two very close to each other. We are not confident about the obtained values for polyoxymethylene. The accuracy for them is questionable because of the low soot content. I tend to agree with the relative ordering of your experimental results.