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Development of a Network Analogy and Evaluation of Mean Beam Lengths for Multidimensional Absorbing/Isotropically Scattering Media

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Based on Hottel's zonal formulation, a network analogy is developed for the analysis of radiative transfer in general multidimensional absorbing/isotropically scattering media. Applying the analogy to the analysis of an isothermal medium and assuming that the incoming and outgoing flux density is homogeneous within the medium, the effect of scattering on the evaluation of mean beam lengths is illustrated. Two concepts of mean beam length, an absorption mean beam length (AMBL) and an extinction mean beam length (EMBL), are introduced and shown to be important for the analysis of radiative transfer in practical systems. Both mean beam lengths differ significantly from the conventional mean beam length in systems of moderate and large optical thickness. Relations between AMBL and EMBL and their limiting behavior are developed analytically. Numerical results for a sphere radiating to its surface and an infinite parallel slab radiating to one of its surfaces are presented to demonstrate quantitatively the mathematical behavior of the two mean beam lengths.

1 Introduction

Radiative transfer in an absorbing/scattering medium is an important aspect in many practical engineering problems such as the modeling of energy transport in combustion chambers and furnaces. Mathematically, an exact solution to the radiative equation of transfer, coupled with the fluid flow and convective heat transfer occurring inside a furnace, can be extremely complicated and therefore unsuitable for practical engineering applications. A great deal of research effort in radiative transfer in the past fifty years has thus been directed toward the development of approximation methods. Sarofim (1986) and Viskanta and Menguc (1987) have given excellent reviews of the various available techniques.

One of the most important concepts utilized in practical radiative transfer calculations, particularly in applications for nongray media, is mean beam length (MBL). Introduced originally by Hottel (1927, 1967), the MBL of a gas volume radiating to a boundary is defined as the radius of an "equivalent" hemisphere that produces a flux to the center of its base equal to the average flux radiated to the area of interest by the actual volume of gas. Since spectral absorption data for all common gaseous species are generally measured in one-dimensional line-of-sight experimental systems, the MBL concept provides an important theoretical link through which the existing one-dimensional spectral absorption data can be applied to radiative transfer calculations in complex multidimensional systems.

In recent years, scattering has been recognized to be important in many particle-laden combustion systems such as fires and pulverized coal furnaces (DeRis, 1978; Wessel, 1985; Menguc and Viskanta, 1986). While it is well known that scattering can have a significant effect on the total radiative heat transfer, its effect on MBL has not yet been established. To the best of the author's knowledge, an approximate expression for the scattering mean beam length in the limit of an optically thin weakly scattering medium (Cartigny, 1986) appears to be the only reported work addressing this important issue. Indeed, the lack of an accurate definition for MBL in an absorbing/scattering medium has led to large uncertainty on the result of many existing furnace calculations (Menguc and Viskanta, 1986; Viskanta and Menguc, 1987).

In this work, a theoretical formulation of MBLs in an absorbing/scattering medium is derived. In section 2, a network analogy for the calculation of radiative transfer in an emitting, absorbing, and isotropically scattering medium is developed. By applying the analogy to an isothermal medium and assuming that the incoming and outgoing flux density is homogeneous, two concepts of MBL, an "absorption" mean beam length (AMBL) and an "extinction" mean beam length (EMBL), are introduced in section 3. Mathematically, AMBL is defined as the radius of a purely absorbing hemisphere with the same absorption coefficient as the medium under consideration, which produces a flux at the center of its base equal to the average heat flux radiated to the area of interest. The concept of AMBL is important in estimating the emission from nongray gaseous species in the presence of scattering particles. EMBL, on the other hand, is defined as the radius of an absorbing/scattering hemisphere with the same extinction coefficient and scattering albedo as the medium under consideration that produces the equivalent heat flux. The concept of EMBL is important in the scaling of scattering/absorbing media. Based on evaluation of AMBL for an absorbing/scattering hemispherical medium radiating to its base, a universal relation between AMBL and EMBL is also presented in section 3. In section 4, exact mathematical expression for the two MBLs are generated for two enclosures, a sphere and an in-

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finite parallel slab. Numerical results are presented to illustrate parametrically the importance of scattering on the evaluation of MBLs. Finally, a conclusion of the present work and some comments on the future direction of research in this general area are presented in section 5.

2 Network Analogy for an Absorbing/Scattering Medium

The analogy between electrical network and radiative transfer in an enclosure with a nonparticipating medium was first developed by Oppenheim (1956). Bevans and Dunkle (1960) extended the idea to include the effect of an isothermal absorbing medium. The extension, however, is quite complicated mathematically and applicable only for a homogeneous isothermal absorbing medium. The practical application for the Bevans and Dunkle formulation is thus limited. Utilizing results generated by the two-flux model, Tong and Tien (1980) noted that the heat transfer in a planar absorbing/scattering medium can be expressed by a network representation. But they did not consider extension to general multidimensional systems. To the best of the author's knowledge, formulation of the network analogy for a general nonisothermal multidimensional enclosure with an absorbing/scattering medium has not appeared in the literature.

For an isotropically scattering and absorbing medium, the set of radiative exchange relations generated by Hottel's zonal method (Hottel and Sarofim, 1967) is a natural basis for the development of the network analogy. Specifically, in an enclosure with N isothermal surface zones and M gas zones, the energy balance on each zone can be written as

$$A_iH_i=\sum_{j=1}^{N}\overline{s_is_j}W_j+\sum_{j=1}^{M}\overline{s_ig_j}W_{g,j} \tag{1}$$

$$4KV_iH_{g,i}=\sum_{j=1}^{N}\overline{g_is_j}W_j+\sum_{j=1}^{M}\overline{g_ig_j}W_{g,j} \tag{2}$$

In the above expressions, H_i and W_i are the irradiation and radiosity (emission plus reflection) of surface A_i while $4H_{g,i}$ and $4KW_{g,i}$ can be interpreted as the incoming and outgoing (emission plus scattering) flux density of a volume element V_i. K is the extinction coefficient. $\overline{s_is_j}$, $\overline{s_ig_j}$, and $\overline{g_ig_j}$ are the direct exchange factors between A_i and A_j, A_i and V_j, and V_i and V_j, respectively. The mathematical expressions for these exchange factors are given by Hottell and Sarofim (1967) and will not be repeated here. In general, they satisfy the following reciprocity and closure relations:

$$\overline{s_is_j}=\overline{s_js_i} \tag{3a}$$

$$\overline{s_ig_j}=\overline{g_js_i} \tag{3b}$$

$$\overline{g_ig_j}=\overline{g_jg_i} \tag{3c}$$

$$\sum_{i=1}^{N}\overline{s_is_j}+\sum_{i=1}^{M}\overline{g_is_j}=A_j \tag{4a}$$

$$\sum_{i=1}^{M}\overline{g_ig_j}+\sum_{i=1}^{N}\overline{s_ig_j}=4KV_j \tag{4b}$$

The parameters W_i, H_i, $W_{g,i}$, and $H_{g,i}$ are related to the surface emissive power E_i and the gaseous emissive power $E_{g,i}$ by

$$W_i=\epsilon_iE_i+(1-\epsilon_i)H_i \tag{5a}$$

$$W_{g,i}=(1-\omega_0)E_{g,i}+\omega_0H_{g,i} \tag{5b}$$

Q_i and $Q_{g,i}$, the net heat transfer from surface A_i and volume V_i, respectively, can be written as

$$Q_i=A_i(W_i-H_i)=\frac{A_i\epsilon_i}{1-\epsilon_i}(E_i-W_i) \tag{6a}$$

$$Q_{g,i}=4V_i(W_{g,i}-H_{g,i})=4KV_i\left(\frac{1-\omega_0}{\omega_0}\right)(E_{g,i}-W_{g,i}) \tag{6b}$$

where ϵ_i is the surface emissivity of A_i and ω_0 is the scattering albedo of the medium.

To develop the necessary equation for the network analogy, the first half of equations (6a) and (6b), together with the reciprocity and closure relations, can be readily combined with equations (1) and (2) to yield

$$Q_i=\sum_{j=1}^{N}\overline{s_is_j}(W_i-W_j)+\sum_{j=1}^{M}\overline{s_ig_j}(W_i-W_{g,j}) \tag{7a}$$

and

$$Q_{g,i}=\sum_{j=1}^{N}\overline{g_is_j}(W_{g,i}-W_j)+\sum_{j=1}^{M}\overline{g_ig_j}(W_{g,i}-W_{g,j}) \tag{7b}$$

Based on equations (6a), (6b), (7a), and (7b), the analogy

Nomenclature

A_i = ith surface area in an enclosure
D = thickness of the parallel slab system
E_i = blackbody emissive power of surface A_i
$E_{g,i}$ = blackbody emissive power of volume $V_{g,i}$
$E_1(x)$ = exponential integral function
$F(\beta)$ = function defined in equation (A9)
$\overline{g_ig_j}$ = volume–volume direct exchange factor between $V_{g,i}$ and $V_{g,j}$
$\overline{g_is_j}$ = volume–gas direct exchange factor between $V_{g,i}$ and A_j
H_i = irradiation into surface A_i
$4H_{g,i}$ = incoming flux density into volume $V_{g,i}$
K = extinction coefficient
L_{ab} = absorption mean beam length
L_e = conventional mean beam length (defined for a pure absorption medium)
L_{ex} = extinction mean beam length
q = heat flux
Q_i = total heat transfer from surface A_i
$Q_{g,i}$ = total heat transfer from volume $V_{g,i}$
R = radius
$\overline{s_is_j}$ = surface–surface direct exchange factor between A_i and A_j
$\overline{s_ig_j}$ = surface–gas direct exchange factor between A_i and V_j
$V_{g,i}$ = ith gas volume in an enclosure
W_i = radiosity from surface A_i
$4KW_{g,i}$ = outgoing flux density from volume $V_{g,i}$
α = angle defined in Fig. A1
β = angle defined in Fig. A1
ϵ_i = emissivity of surface A_i
θ = angle defined in Fig. A1
θ_1 = angle defined by equation (A10a)
θ_2 = angle defined by equation (A10a)
ϕ = azimuthal angle used in equations (A6), (A7), and (A9)
ϕ_m = angle defined by equation (A11)
ω_0 = scattering albedo

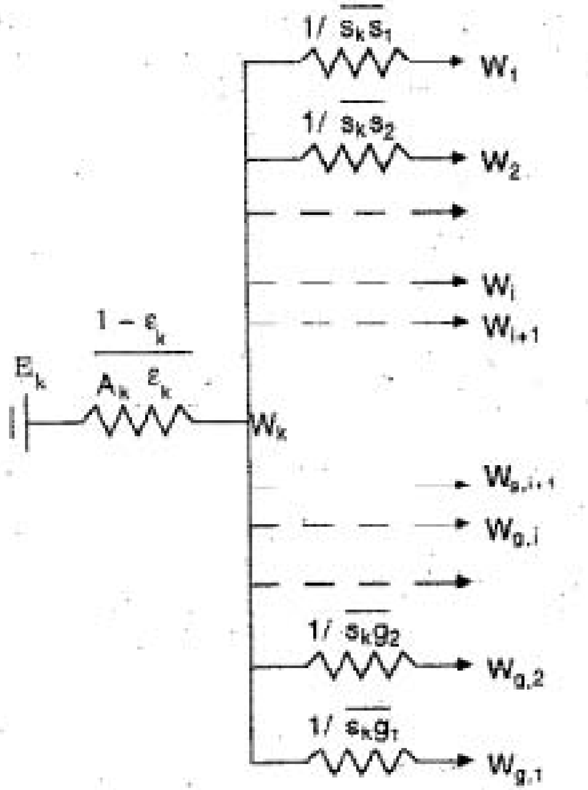

Fig. 1(a) Network representation for a surface element A_k

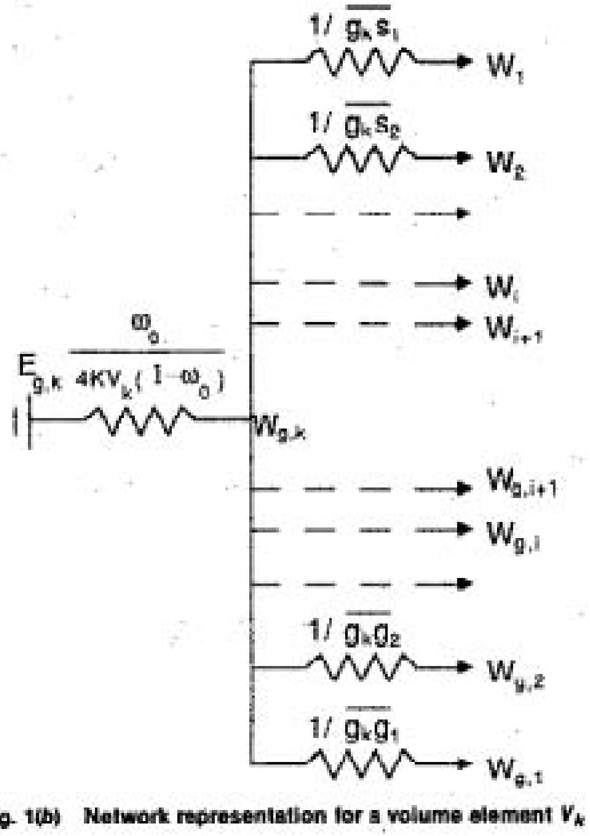

Fig. 1(b) Network representation for a volume element V_k

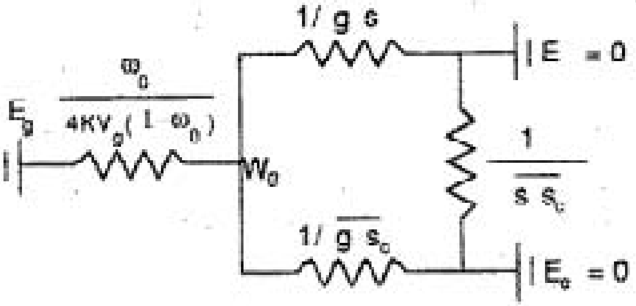

Fig. 2 Network representation for the mean beam length calculation

between electrical network and radiative heat transfer in an absorbing/scattering medium is quite apparent. Specifically

$$E_i = \text{internal potential of surface } A_i$$

$$W_i = \text{external potential of surface } A_i$$

$$\frac{1-\epsilon_i}{A_i \epsilon_i} = \text{internal resistance at surface } A_i$$

$$E_{g,i} = \text{internal potential of volume } V_i$$

$$W_{g,i} = \text{external potential of volume } V_i$$

$$\frac{\omega_0}{4KV_i(1-\omega_0)} = \text{internal resistance at volume } V_i$$

$$1/\overline{s_i s_j} = \text{resistance between surfaces } A_i \text{ and } A_j$$

$$1/\overline{s_i g_j} = \text{resistance between surfaces } A_i \text{ and } V_j$$

$$1/\overline{g_i g_j} = \text{resistance between surfaces } V_i \text{ and } V_j$$

Schematic representations of the network analogy for a surface element A_i and a volume element V_i are shown in Figs. 1(*a*) and 1(*b*), respectively. Note that in contrast to Bevans and Dunkle (1960), the present network analogy does not require the restrictive assumption of an isothermal gas volume. It can be applied to any enclosure containing an absorbing/isotropically scattering medium.

3 Formulation of Mean Beam Lengths

In the formulation of mean beam lengths, the problem of interest is to determine the radiative heat flux incident on an area A from a homogeneous isothermal gas volume (with volume V_g, emissive power E_g, extinction coefficient K, and scattering albedo ω_0). The area A, in general, is a part of the total boundary of the enclosure. Let A_o be the remaining area of the enclosure and assuming that H_g and W_g are uniform in the gas volume, the network analogy is applicable with an equivalent network as shown in Fig. 2. The corresponding network equations can be readily solved to yield the following heat flux expression at surface A:

$$q = \frac{(E_g \overline{sg}/A)(1-\omega_0)}{1 - \frac{\omega_0 \overline{gg}}{4KV_g}} \tag{8}$$

3(*a*) Concept of Absorption Mean Length (AMBL). The absorption mean beam length (AMBL), L_{ab}, is defined as the radius of a hemispherical volume of purely absorbing gas (with the same temperature and absorption coefficient as that of the actual mixture), which produces a flux to the center of its base equal to the acutal heat flux. Mathematically, L_{ab} is given by the relation

$$1 - e^{-(1-\omega_0)KL_{ab}} = \frac{(1-e^{-KL_e})(1-\omega_0)}{1 - \frac{\omega_0 \overline{gg}}{4KV_g}} \tag{9}$$

where the exchange factor $\overline{sg}$ is written in term of the conventional mean beam length L_e as

$$\frac{\overline{sg}}{A} = 1 - e^{-KL_e} \tag{10}$$

The concept of AMBL is useful when it is necessary to estimate the effect of scattering particles on the emission of a second species (e.g., the evaluation of the strength of a gaseous absorption band in the presence of scattering particles in furnace calculation). Note that the mean beam length expression considered by Cartigny (1986) is the absorption mean beam length.

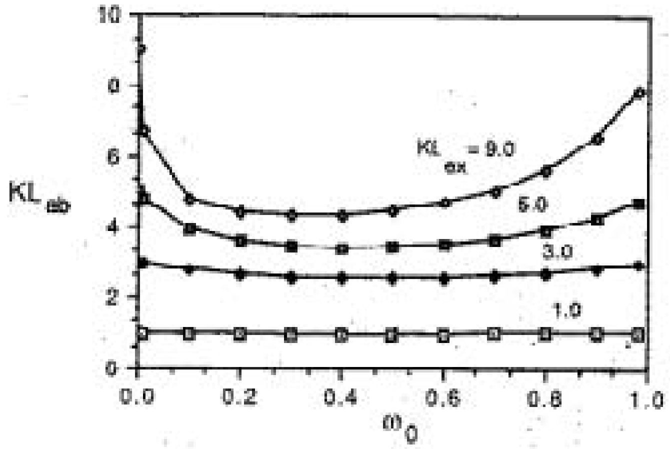

Fig. 3 Universal relation between KL_{ab}, KL_{ex}, and ω_0

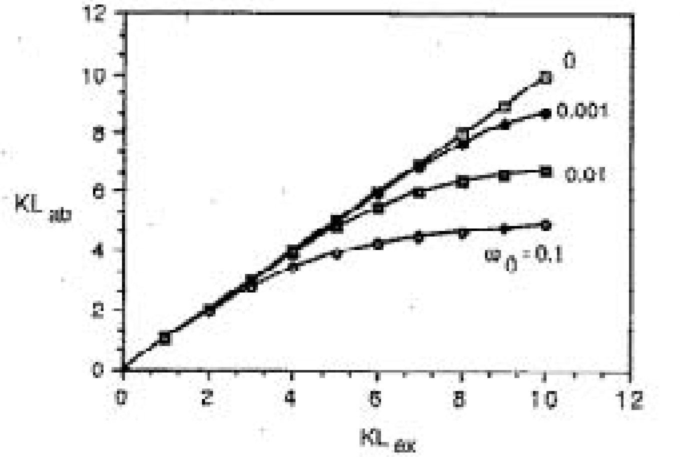

Fig. 4 Relation between KL_{ab} and KL_{ex} in the weak scattering limit

3(*b*) Concept of Extinction Mean Bean Length (EMBL). The extinction mean beam length (EMBL), L_{ex}, is defined as the radius of a hemispherical volume of scattering and absorbing gas (with the same temperature, extinction coefficient, and scattering albedo as that of the actual mixture), which produces a flux to the center of its base equal to the acutal heat flux. This concept is important for the scaling of a mixture in which the effects of absorption and scattering are not readily separable (e.g., the assessment and comparison of optical thicknesses for two geometrically dissimilar sooty flames). Unlike L_e and L_{ab}, a closed-form expression for L_{ex} is not available for an arbitrary enclosure since the heat flux to the center of the base of an absorbing/scattering hemisphere cannot be expressed in closed form. A universal relation between L_{ab} and L_{ex}, however, can be readily generated based on the expression of L_{ab} for a hemispherical volume of gas radiating to the center of its base. Since L_{ex} for a hemisphere radiating to its base is simply its radius, L_{ex} and L_{ab} for an arbitrary enclosure are related by

$$1-e^{-(1-\omega_0)KL_{ab}}=\frac{(1-e^{-KL_{ex}})(1-\omega_0)}{1-\omega_0\left(\frac{\overline{gg}}{4KV_g}\right)_{\text{hemisphere}}} \tag{11}$$

where $(\overline{gg}/4KV_g)_{\text{hemisphere}}$ is the self-exchange factor per unit volume for a hemispherical volume of gas with optical radius KL_{ex}. Specifically, the evaluation of EMBL for an arbitrary enclosure requires first the evaluation of AMBL based on equation (9) and then the evaluation of EMBL based on equation (11).

The evaluation of $(\overline{gg}/4KV_g)_{\text{hemisphere}}$, which has not appeared in the literature, is presented in Appendix A. Numerical values for KL_{ab} for different values of KL_{ex} and ω_0 are presented in Fig. 3. It can be readily observed that, in general, $L_{ab} \le L_{ex}$. In a hemispherical geometry, the primary effect of scattering is to reduce heat transfer. A larger absorbing/scattering hemisphere is required to generate the same heat flux to the center of its base as that of a purely absorbing hemisphere.

3(*c*) Limiting Behavior of L_{ex} and L_{ab}. In the optically thin limit ($KL_{ex}\to 0$) and independent of ω_0, equations (9) and (11) are reduced to

$$L_e = L_{ex} = L_{ab} \tag{12}$$

Based on numerical data presented in Fig. 3, L_{ex} and L_{ab} are essentially identical for $KL_{ex} \le 1.0$.

Physically, the three mean beam lengths are also expected to be identical in the limit of no scattering ($\omega_0 = 0$). Numerical data, however, show that the convergence of L_{ab} to L_{ex} is quite slow in the optically thick limit. In Fig. 4, the relation between L_{ab} and L_{ex} in the weak-scattering limit ($\omega_0 \le 0.1$) is presented. Note that even for a "small" scattering albedo (say, $\omega_0 = 0.01$), the difference between L_{ex} and L_{ab} can be quite substantial in systems with moderate and large optical thicknesses. Equation (12), in the more restrictive optically thin weakly scattering limit ($KL_{ab}\to 0$, $\omega_0 \to 0$), is also deduced by Cartigny (1986).

In the strongly scattering limit ($\omega_0 \to 1.0$), equation (9) can be simplified to yield

$$KL_{ab}=\frac{\overline{sg}/A}{1-\frac{\overline{gg}}{4KV_g}} \tag{13}$$

and the relation between L_{ab} and L_{ex} becomes

$$KL_{ab}=\frac{(1-e^{-KL_{ex}})}{1-\omega_0\left(\frac{\overline{gg}}{4KV_g}\right)_{\text{hemisphere}}} \tag{14}$$

Note that AMBL and EMBL are both nonzero even in the pure scattering limit ($\omega_0 = 1.0$). Similar to the conventional MBL, L_e, L_{ab}, and L_{ex} are fundamental geometric parameters of the enclosure.

If A is the total surface to the enclosure (i.e., $A_c = 0$ in Fig. 2), equation (13) can be further simplified (using the reciprocity and closure relations) to yield

$$L_{ab}=\frac{4V_g}{A} \tag{13a}$$

which is identical to the optically thin limiting expression for L_e. Physically, this result is not surprising because from the pure absorption consideration, a pure scattering medium is optically thin. It is important to emphasize that equation (13*a*) holds for all strongly scattering media independent of optical thickness. L_{ex}, in the pure scattering limit, must still be evaluated using equation (14).

4 Numerical Examples

To illustrate quantitatively the effect of scattering on AMBL and EMBL, numerical values are tabulated for two specific geometries, a sphere and an infinite parallel slab. These two cases are selected because many practical furnaces can be approximated by these geometries and closed-form expressions for the necessary exchange factors $\overline{sg}/A$ and $\overline{gg}/(4KV_g)$ are available from standard references (Hottel and Sarofim, 1967).

For an absorbing-scattering spherical medium radiating to its surface, L_{ab} is given by

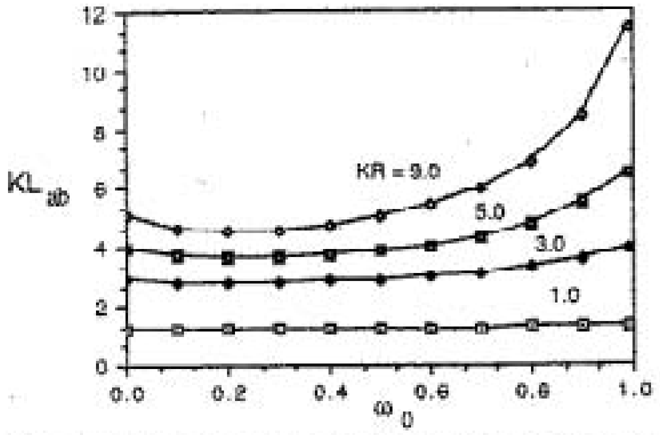

Fig. 5(a) Absorption mean beam length for a sphere radiating to its surface for different optical radii (KR) and albedos (ω_0)

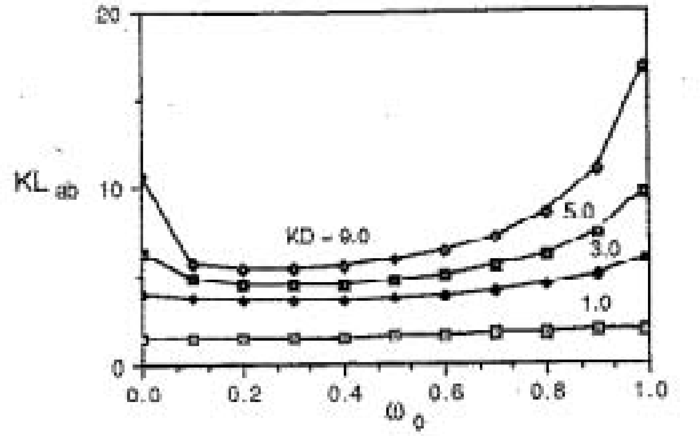

Fig. 6(a) Absorption mean beam length for an infinite parallel slab radiating to one of its surfaces for different optical slab thicknesses (KD) and albedos (ω_0)

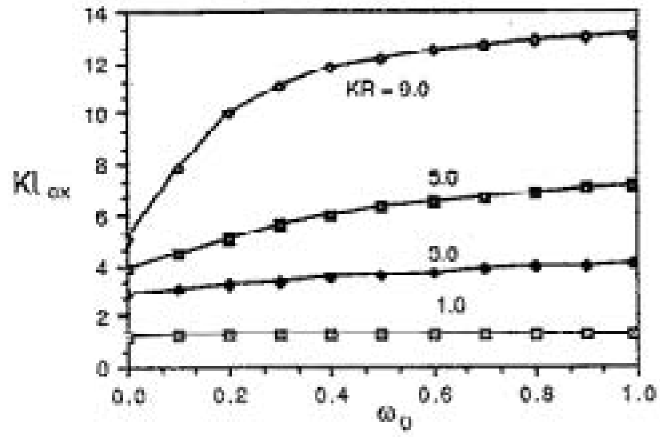

Fig. 5(b) Extinction mean beam length for a sphere radiating to its surface for different optical radii (KR) and albedos (ω_0)

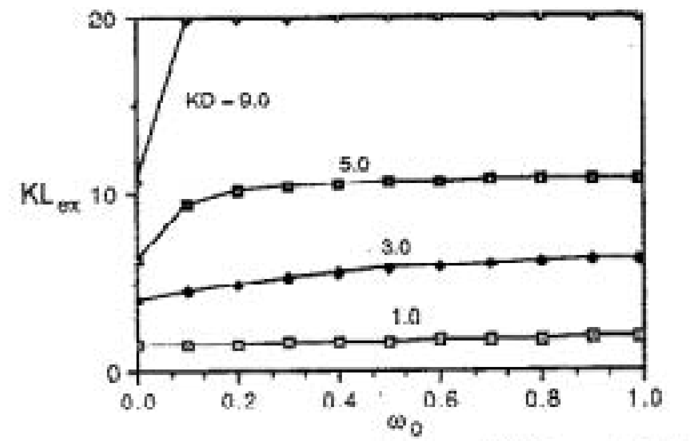

Fig. 6(b) Extinction mean beam length for an infinite parallel slab radiating to one of its surfaces for different optical slab thicknesses (KD) and albedos (ω_0)

$$1-e^{-(1-\omega_0)KL_{ab}}=\frac{\frac{\overline{sg}}{A}(1-\omega_0)}{1-\omega_0\left(1-\frac{3}{4KR}\frac{\overline{sg}}{A}\right)} \tag{14}$$

where

$$\frac{\overline{sg}}{A}=1-\frac{1}{2(KR)^2}\left(1-(2KR+1)e^{-2KR}\right) \tag{15}$$

with R being the radius of the sphere. Numerical values of AMBL and EMBL for different KR and ω_0 are presented in Figs. 5(a) and 5(b). While L_{ex}, in general, increases monotonically with ω_0, L_{ab} first decreases and then increases with increasing ω_0. Physically, the slight decrease in L_{ab} with ω_0 at small albedo can be attributed to the slight enhancement of heat transfer by scattering in an optically thick medium. As ω_0 increases, the effect of extinction by scattering becomes dominant and L_{ab} increases. A comparison between Figs. 5(a) and 5(b) also shows that, in general, L_{ex} is much greater than L_{ab}, particularly in the limit of large albedo.

For a parallel slab of absorbing-scattering medium radiating to one of its two bounding surfaces, L_{ab} is given by

$$1-e^{-(1-\omega_0)KL_{ab}}=\frac{\frac{\overline{sg}}{A}(1-\omega_0)}{1-\omega_0\left(1-\frac{1}{2KD}\frac{\overline{sg}}{A}\right)} \tag{16}$$

where

$$\frac{\overline{sg}}{A}=1-2E_3(KD) \tag{17}$$

with D being the thickness of the slab and $E_3(x)$ the exponential integral function (Hottel and Sarofim, 1967). Numerical results of L_{ex} and L_{ab} for a slab are shown in Figs. 6(a) and 6(b). Their qualitative behavior is similar to that for the sphere presented in Figs. 5(a) and 5(b).

5 Conclusions

A network analogy is developed for the analysis of radiative transfer in an absorbing and isotropically scattering medium. Based on a network analysis for an isothermal medium and assuming that the incoming and outgoing flux density are homogeneous, the traditional concept of mean beam length is extended to include the effect of scattering. Two concepts of mean beam length (an absorption mean beam length, AMBL, and an extinction mean beam length, EMBL) are shown to be useful in characterizing the radiative transfer in a scattering medium. Based on an analysis of a hemispherical absorbing/scattering medium, a universal relation between AMBL and EMBL is developed. Numerical values for AMBL and EMBL for two enclosures are presented to show that the two mean beam lengths differ significantly from each other and also from the conventional mean beam length in systems with moderate or large optical thickness. The use of the conventional definition of mean beam length in general absorbing/scattering media can thus lead to significant error, except in the optically thin limit.

The general behavior of AMBL and EMBL is illustrated by numerical results generated by two specific systems, a spherical medium radiating to its surface and an infinite parallel slab radiating to one of its surfaces. For a fixed extinction coefficient and physical dimension, EMBL is shown to increase monotonically with scattering albedo while AMBL first decreases and then increases with scattering albedo.

Since an isothermal absorbing/scattering medium can, in general, have nonuniform incoming and outgoing flux density, expressions for AMBL and EMBL developed in the present work are approximate. Additional exact numerical calculations must also be carried out to develop a more precise relation between the two mean beam lengths and their mathematical behavior. Systems with different geometric shapes must be analyzed in order to generate quantitative relations, which are useful for practical applications. These efforts are currently under way and the results will be presented in future publications.

References

Bevans, J. T., and Dunkle, R. V., 1960, "Radiative Exchange Within an Enclosure," ASME JOURNAL OF HEAT TRANSFER, Vol. 82, pp. 1-19.

Cartigny, J. D., 1986, "Mean Beam Length for a Scattering Medium," *Heat Transfer—1986*, C. L. Tien, V. P. Carey, and J. K. Farrell, eds., Vol. 2, pp. 769-772.

DeRis, J., 1978, "Fire Radiation—A Review," *The Seventeenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, PA, pp. 1003-1016.

Hottel, H. C., 1927, *Transaction American Institute of Chemical Engineers*, Vol. 19, p. 173.

Hottel, H. C., and Sarofim, A. F., 1967, *Radiative Transfer*, McGraw-Hill, New York.

Menguc, M. P., and Viskanta, R., 1986, "An Assessment of Spectral Radiative Transfer Predictions for a Pulverized Coal-Fired Furnace," *Heat Transfer—1986*, C. L. Tien, V. P. Carey, and J. K. Farrell, eds., Vol. 2, pp. 815-820.

Oppenheim, A. K., 1956, "Radiative Analysis by the Network Method," *Trans. ASME*, Vol. 78, pp. 725-735.

Sarofim, A. F., 1986, "Radiative Heat Transfer in Combustion: Friend or Foe," *The Twenty-First Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, PA, pp. 1-23.

Tong, T. W., and Tien, C. L., 1980, "Resistance-Network Representation of Radiative Heat Transfer With Particulate Scattering," *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 24, pp. 491-503.

Viskanta, R., and Menguc, M. P., 1987, "Radiation Heat Transfer in Combustion Systems," *Progress in Energy and Combustion Science*, Vol. 13, pp. 97-160.

Wessel, R. A., 1985, "Effective Radiative Properties for Particle-Gas Mixtures," *Heat Transfer in Fire and Combustion Systems*, C. K. Law, Y. Jaluria, W. W. Yuen, and K. Miyasaka, eds., pp. 239-249.

APPENDIX

The evaluation of $gg/4KV_g$ for a hemispherical volume of gas is presented in this section. Let A_1 and A_2 be the hemispherical surface and the base surface, respectively; equations (4*a*) and (4*b*) can be readily utilized to yield

$$\frac{\overline{gg}}{4KV_g} = 1 - \frac{\overline{gs_1}}{4KV_g} - \frac{\overline{gs_2}}{4KV_g} \tag{A1}$$

where the two surface–gas exchange factors can be written as

$$\frac{\overline{s_1 g}}{A_1} = 1 - \frac{\overline{s_1 s_1}}{A_1} - \frac{\overline{s_1 s_2}}{A_1} \tag{A2}$$

and

$$\frac{\overline{s_2 g}}{A_2} = 1 - \frac{\overline{s_2 s_1}}{A_2} \tag{A3}$$

The evaluation of $\overline{gg}/4KV_g$ thus requires the evaluation of the two surface exchange factors $\overline{s_1 s_1}$ and $\overline{s_1 s_2}$.

The relevant geometry and coordinate system utilized in the evaluation of $\overline{s_1 s_2}$ is shown in Fig. A1. For the differential areas dA_1 and dA_2 as shown, the differential exchange factor $\overline{ds_1 ds_2}$ is given by

$$\overline{ds_1 ds_2} = dA_1 dA_2 \frac{\cos\theta \cos\alpha e^{-Kd}}{\pi d^2} \tag{A4}$$

where d is the distance between dA_1 and dA_2 and θ and α are as defined in Fig. A1.

Mathematically, it can be readily shown that a direct numerical integration based on a Cartesian coordinate is ineffective in the evaluation of $\overline{s_1 s_1}$ and $\overline{s_1 s_2}$ because the integrand becomes singular as $d \to 0$. To overcome this difficulty, equation (A4) is rewritten in terms of a differential solid angle $d\omega_2$ as

$$\overline{ds_1 ds_2} = dA_1 d\omega_2 \frac{\cos\theta e^{-Kd}}{\pi} \tag{A5}$$

where $d\omega_2$ is given by

$$d\omega_2 = \frac{dA_2 \cos\alpha}{d^2} = \sin\theta d\theta d\phi \tag{A6}$$

with ϕ being the azimuthal angle measured on the surface

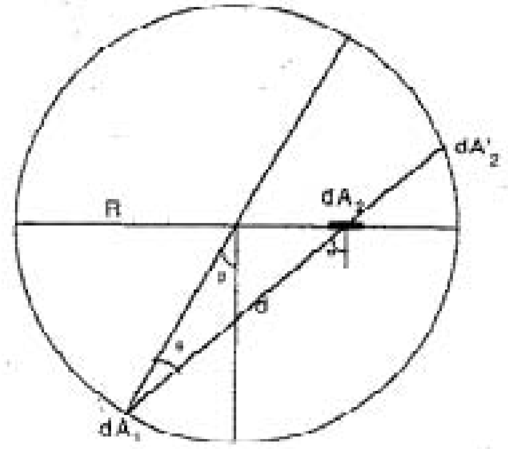

Fig. A1 Geometry and coordinate system for the evaluation of $(gg/4KV_g)_{hemisphere}$

Table 1 Numerical values for the various exchange factors for a hemispherical gas volume

KR	$\overline{s_1 s_1}/A_1$	$\overline{s_1 s_2}/A_1$	$\overline{gg}/4KV_g$
0.00	0.5000e+00	0.5000e+00	0.0000e+00
0.01	0.4944e+00	0.4962e+00	0.7716e-02
0.05	0.4727e+00	0.4813e+00	0.2858e-01
0.10	0.4471e+00	0.4633e+00	0.5266e-01
0.20	0.4006e+00	0.4301e+00	0.1029e+00
0.30	0.3597e+00	0.3999e+00	0.1490e+00
0.40	0.3241e+00	0.3724e+00	0.1917e+00
0.50	0.2920e+00	0.3474e+00	0.2301e+00
0.60	0.2641e+00	0.3245e+00	0.2665e+00
0.70	0.2392e+00	0.3037e+00	0.2998e+00
0.80	0.2171e+00	0.2845e+00	0.3308e+00
0.90	0.1976e+00	0.2670e+00	0.3597e+00
1.00	0.1802e+00	0.2510e+00	0.3867e+00
2.00	0.8044e-01	0.1458e+00	0.5770e+00
5.00	0.1740e-01	0.5283e-01	0.7935e+00
10.00	0.4681e-02	0.2382e-01	0.8914e+00