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Determination of Optical Properties by Two-Dimensional Scattering

W. W. Yuen* and A. Ma†
University of California at Santa Barbara, Santa Barbara, California 93106
and
I. C. Hsu‡ and G. R. Cunnington Jr.‡
Lockheed Missiles & Space Company, Inc., Palo Alto, California 94304

Nomenclature

a = absorption coefficient
D = thickness of absorbing/scattering sample
ds_1g = source-to-volume exchange factor vector
ds_1g_i = exchange factor between source and volume zone V_i
$\overline{gg}$ = volume-volume exchange factor matrix
g_ig_j = exchange factor between volume zones V_i and V_j
I_0 = flux density of source
K = extinction coefficient
L = half-width of absorbing/scattering sample
Q_2 = heat transfer to detector
S = error sum defined by Eq. (12)
$S_n(x)$ = function defined in Ref. 5
s_2g = detector-to-volume exchange factor vector
$(s_2g)^T$ = transpose of s_2g
s_2g_i = exchange factor between $2W$ and V_i
t_{ij} = distance parameter defined by Eq. (9)
$\bar{V}$ = volume matrix
V_i = volume zonal element
W = half-width of detector window opening
W_{gi} = radiosity of volume zone V_i
$2W$ = detector width
x_i = coordinate of V_i or A_i
x_2 = coordinate measured along detector $2W$
z_i = coordinate of V_i or A_i
τ = transmissivity
ω = scattering albedo

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*Professor. Member AIAA.
†Graduate Student.
‡Technical Staff, Thermal Science Laboratory. Members AIAA.

I. Introduction

THE importance of radiative heat transfer in the assessment of insulation materials for aerospace applications is well known. For practical design and performance evaluation of many engineering components, the optical properties of insulation materials are important parameters that must be accurately determined.

Theoretically, the optical properties of material are related to its index of refraction and various geometric parameters (particle/fiber or void size distribution and number density) and can, in principle, be computed by Mie Theory. A number of such computations have been reported.[1,2] But due to the variation in the index of refraction and the geometric complexity, such calculations are quite tedious and have uncertain accuracy. Indeed, the most common approach currently utilized by the aerospace industry is to determine optical properties of materials empirically by in situ optical transmission measurement.

Fundamentally, the basis of the optical transmission measurement is the Beer-Lambert law. For a slab of thickness D, the transmissivity τ is related to the absorption coefficient a by the relation

$$\tau = e^{-aD} \tag{1}$$

Assuming that a remains constant within the material, repeated measurements of τ at different D provide a set of statistically reliable data from which a can be determined. Experiments performed at different wavelengths and slab temperatures will determine the spectral and temperature dependence of a.

But while the optical transmission measurement is quite effective in determining a for nonscattering materials, it is not applicable for scattering materials because of the multidimensional effect of scattering. For example, consider the transmission of a collimated line source through a two-dimensional planar slab as shown in Fig. 1. Data obtained by Lockheed for an aerospace insulation material, LI900, are shown in Fig. 2. It is apparent that the Beer-Lambert law is inadequate as it fails to account for the scattering contribution to τ. For a medium with extinction coefficient K and scattering albedo ω, τ is a function of the optical depth KD, the optical width KL, ω, and the optical width of the detecting area KW. Until now, the lack of a reliable functional relation between τ, KD, KL, KW, and ω is probably the primary factor leading to the existing large uncertainty in optical properties for most insulating scattering materials.

The objective of the present work is to show that based on a recently developed network formulation,[3] the relation between τ and the optical properties of a two-dimensional scattering material can be determined numerically with little complexity. Based on these results, two-dimensional transmission data are inverted to determine "best-fit" values for the optical properties. For simplicity, this procedure is illustrated only for an isotropically scattering material and a two-dimensional

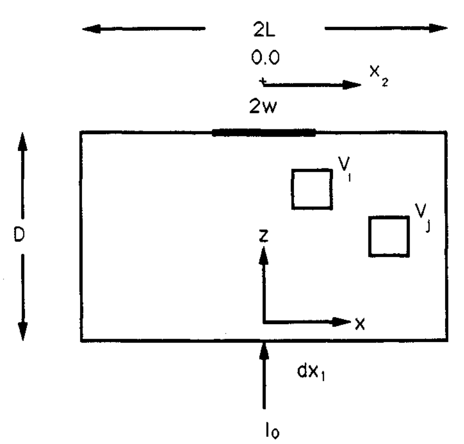

Fig. 1 Geometry of the two-dimensional planar scattering system.

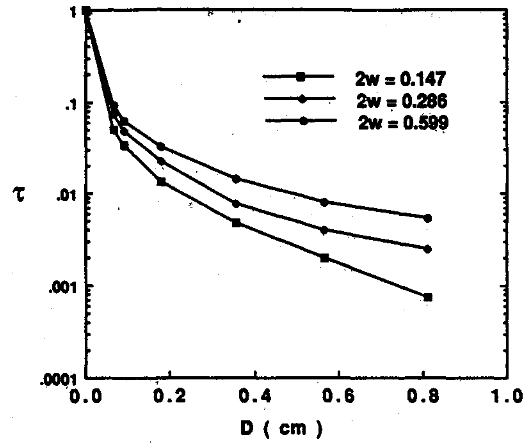

Fig. 2 Transmission data for LI900.

planar slab in this work. Extension to two-dimensional cylindrical geometry and anisotropically scattering materials will be presented in future publications.

II. Mathematical Model

Consider the zonal arrangement for a two-dimensional slab as shown in Fig. 1. In a transmission measurement, a line source of infinitesimal width dx_1 and flux density I_0 is incident on a finite planar slab of width $2L$ and thickness D. A detector of width $2W$ (with $W < L$) is used to measure the transmitted radiation.

Based on the Hottel zonal method[4] and standard network analysis technique,[3] an energy balance at a volume V_i in Fig. 1 can be written as

$$4K \frac{V_i}{\omega} W_{gi} = \text{d}s_1g_iI_0 + \sum_{j=1}^{N} (g_ig_j)W_{gj} \qquad (2)$$

with W_{gi} and W_{gj} being the net outgoing radiosity from volume V_i and V_j, respectively. $\text{d}s_1g_i$ is the source-to-volume exchange factor and g_ig_j is the exchange factor from volume i to volume j. Since the objective of the analysis is to determine the effective transmission of the incident energy beam, all emissions (from the medium, the detector and the surrounding boundary) are neglected in the development of Eq. (2). Equation (2) can be readily inverted to yield

$$\mathbf{W_g} = \left[4K\frac{\bar{V}}{\omega} - \overline{gg}\right]^{-1} \text{d}s_1\mathbf{g}I_0 \qquad (3)$$

where $\mathbf{W_g}$ and $\text{d}s_1\mathbf{g}$ are N-component radiosity and source-to-volume exchange factor vectors, respectively. $\bar{V}$ and $\overline{gg}$ are the respective $N \times N$ volume matrix and volume-volume exchange factor matrix.

The net heat transfer Q_2 to the detector $2W$ in Fig. 1 is

$$Q_2 = I_0\text{d}s_1s_2 + \sum_{j=1}^{N} (s_2g_j)W_{gj} \qquad (4)$$

with $\text{d}s_1s_2$ being the source-to-detector exchange factor, and s_2g_j being the volume-to-detector exchange factor. Combining Eqs. (3) and (4), the effective transmissivity of the medium becomes

$$\tau = \frac{1}{\text{d}x_1}[\text{d}s_1s_2 + \omega(s_2\mathbf{g})^T[4K\bar{V} - \omega\overline{gg}]^{-1}(\text{d}s_1\mathbf{g})] \qquad (5)$$

with $(s_2\mathbf{g})^T$ being the transpose of the vector, $s_2\mathbf{g}$. In the above equations, various exchange factors are given by

$$s_2g_i = \int_{V_i}\int_{2W} \frac{K\text{d}x_i\text{d}z_i\text{d}x_2(D - z_i)S_2(Kt_{i2})}{t_{i2}^2} \qquad (6)$$

$$\text{d}s_1g_i = \text{d}x_1\int_{V_i} \frac{K\text{d}x_i\text{d}z_iz_iS_2(Kt_{i1})}{t_{i1}^2} \qquad (7)$$

and

$$g_ig_j = \int_{V_i}\int_{V_j} \frac{K^2\text{d}x_i\text{d}z_i\text{d}x_j\text{d}z_jS_1(Kt_{ij})}{t_{ij}} \qquad (8)$$

with

$$t_{ij} = [(x_i - x_j)^2 + (z_i - z_j)^2]^{1/2} \qquad (9)$$

and $S_n(Kt)$ defined in Ref. 5. For a collimated source, the exchange factor between the source and the detector, $\text{d}s_1s_2$, is given by

$$\text{d}s_1s_2 = \text{d}x_1e^{-KD} \qquad (10)$$

while for a diffuse source, the exchange factor becomes

$$\text{d}s_1s_2 = \text{d}x_1\int_{2W} \frac{\text{d}x_2D^2S_3(Kt_{21})}{t_{21}^3} \qquad (11)$$

An important step in the evaluation of Eq. (5) is the numerical tabulation of the various exchange factors. It can be readily shown that with the established analytical and numerical properties of $S_n(Kt)$,[5] many of the required numerical integrals can be simplified in many cases to single integrals, and evaluated with little effort. The detailed mathematical procedure is presented in an earlier publication.[6]

III. Experimental Procedure and Data Reduction

The experimental approach is straightforward. Specifically, transmission measurements for a collimated source at a specific wavelength (He-Ne Laser at 0.638 μm) are made with samples of different width, depth, and detectors of different window openings. The two-dimensional planar effect is simulated by shaping the incoming beam as a line source in the lengthwise direction of the sample. The detector opening is a rectangle with a finite width and a length identical to that of the sample. Standard least-square curve-fitting procedure is used to determine the best-fit values of the two unknown parameters, K and ω.

It is interesting to note that in the standard one-dimensional approach based on the Beer-Lambert law, measurements at moderate and large optical thicknesses are generally required to yield statistically reliable estimates of the extinction coefficient. Measurements for media with large optical thicknesses, however, are often unreliable because of the extremely small value of the transmissivity. The accuracy of the one-dimensional approach in determining K is thus naturally limited.

The experimental data presented in this work, on the other hand, show that the transmissivity can vary significantly with the detector window opening, even in samples with small optical depth. Since the measured value of the transmissivity is high, the relative error of measurement is small. The present approach should be a reliable technique in measuring optical properties, particularly for media with moderate or large scattering albedo.

Numerical results based on LI900 data are used as an example to demonstrate the unique combination of this experimental procedure and Eq. (5) to resolve optical properties. Two factors, however, limited the accuracy of the present calculation. First, LI900 is known to be a highly anisotropic forward-scattering material. Based on a Mie calculation for the known fiber size distribution of the test sample, for example, the backward-scattering fraction was estimated to be 0.173.[7] The backward-scattering fraction for an isotropically scattering medium is expected to be 0.5. Second, the optical

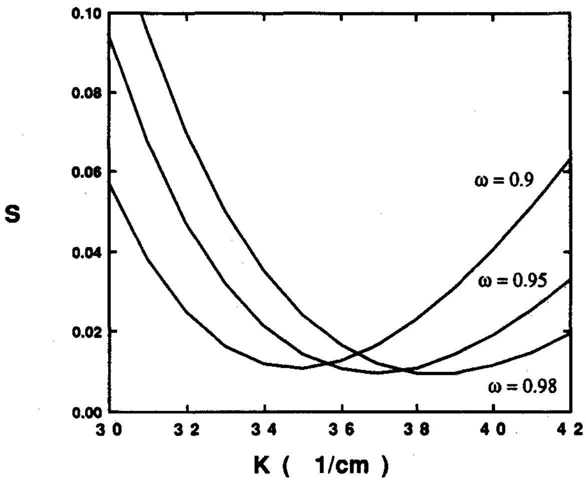

Fig. 3 Error sum calculated for different K and ω.

width of the scattering sample (KL) was not recorded in the experiment, therefore numerical data for an one-dimensional slab ($KL \to \infty$) are used in the calculation.

An initial estimate for the extinction coefficient K is obtained by interpolating the LI900 transmission data in the optically thin limit. For each assumed K and ω, an error sum is calculated by

$$S = \sum_{\text{data}} (\tau_{\text{measure}} - \tau_{\text{numerical}})^2 \tag{12}$$

with the summation extended to all measured data points. The behavior of S for different ω and K is shown in Fig. 3. It is apparent that the present approach is numerically stable. It yields a unique least-square prediction for $\omega = 0.95$ and $K = 37 \text{ cm}^{-1}$. A Mie calculation, based on the known fiber size distribution of the test sample, yields $\omega = 1.0$, $K = 148 \text{ cm}^{-1}$, and $b = 0.173$. The discrepancy in K can probably be attributed to the anisotropic scattering effect which is not included in the model and the uncertainty of the index of refraction utilized in the Mie calculation. Nevertheless, the agreement in ω and the ability of the model to predict the qualitative behavior of the transmissivity suggest that the current mathematical model and experimental approach is a promising procedure for the determination of optical properties. It is also apparent that this procedure can be readily extended to applications for anisotropically scattering materials.

IV. Conclusions and Future Works

A unique experimental and numerical procedure is shown to be effective in predicting optical properties for a two-dimensional absorbing and isotropically scattering medium. Analysis of some limited data generated for a fibrous insulation material (LI900) shows the potential of present approach.

Additional experiments are needed to verify the statistical reliability and sensitivity of the proposed experimental technique. Since most insulation materials scatter anisotropically, an extension of the numerical procedure to general absorbing, anisotropically scattering media is required. The recently developed generalized zonal method[8] appears to be ideally suited for the extension. These efforts are currently underway and results will be reported in future publications.

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Transient Laminar Forced Convection From a Circular Cylinder Using a Body-Fitted Coordinate System

Yue-Tzu Yang,* Cha'o-Kuang Chen,† and Sang-Ru Wu‡
National Cheng-Kung University, Tainan, Taiwan, Republic of China

Introduction

THE problem of steady flow and heat transfer characteristics around a circular cylinder has been widely investigated. At the situation of Reynolds number $5 < Re < 40$ there exists a pair of steady and symmetric vortices right behind the cylinder. When $Re > 40$, the flow begins to have an unsteady motion. The vortices are oscillating periodically and flowing in the downstream direction. Thoman and Szewczyk,[1] Gresho et al.,[2] and Hwang et al.[3] studied the time-dependent flow characteristics of a circular cylinder. Jordan and Fromm,[4] Lin et al.,[5] Patel,[6] Lecointe and Piquet,[7] and Smith and Brebbia[8] have presented their numerical results for the unsteady incompressible viscous flow past a circular cylinder.

Jain and Goel,[9] McAdam,[10] Kramers,[11] Van der Hegge Zijnen,[12] and Tsubouchi and Masuda[13] discussed the development of the vortex shedding behind a circular cylinder. Karniadakis[14] used the spectral element method to solve forced convection heat transfer from an isolated cylinder in crossflow for Reynolds numbers up to 200. In the present paper, the problem of transient laminar forced convection from a horizontal isothermal cylinder is studied. The Navier-Stokes equations and the energy equation for an unsteady incompressible fluid flow are solved by using a body-fitted orthogonal coordinate system and spline alternating direction implicit (SADI) method, described by Rubin and Graves.[15] The computed results are compared with previous experimental correlations and numerical results. Although there are many investigations concerning this subject, the main contribution of the present paper is that a different numerical technique is applied and the results are extended to $Re = 500$.

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*Associate Professor, Department of Mechanical Engineering.
†Professor, Department of Mechanical Engineering.
‡Graduate Student, Department of Mechanical Engineering.